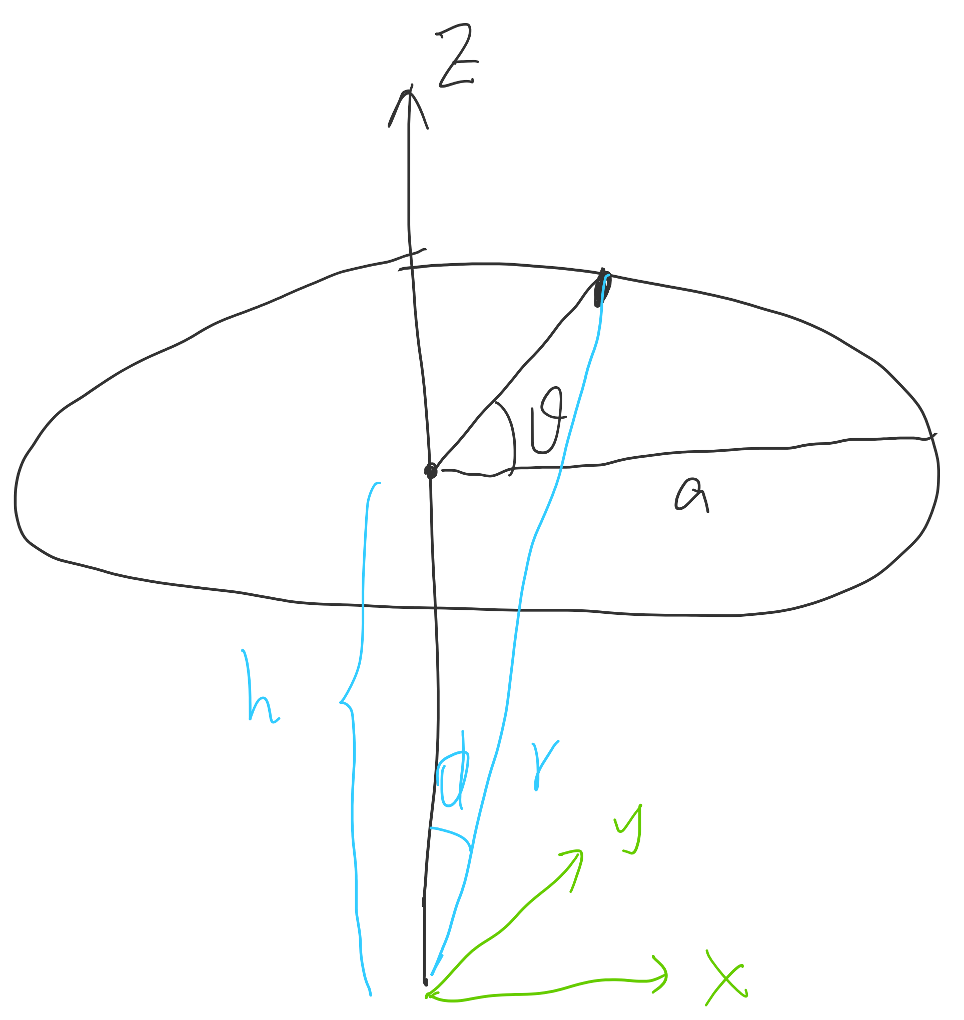
Vio from scratch IMU

Sunday, March 15, 2020 2:00 PM



$r = (a\cos\theta,\ a\sin\theta,\ h)^T$ This is the coordinate of the particle

$\dot r = (-a\dot\theta\sin\theta,\ a\dot\theta\cos\theta,\ 0)^T$ θ is changing, a, h are static.

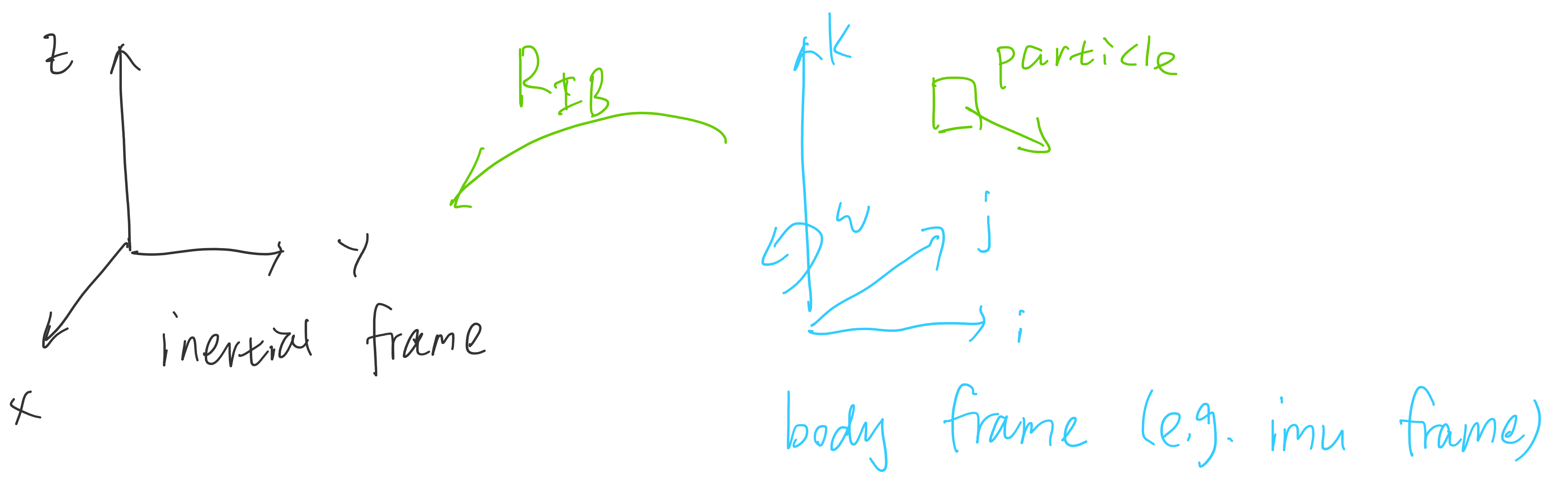
$$= \begin{bmatrix} 0 & -\dot\theta & 0 \\ \dot\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a\cos\theta \\ a\sin\theta \\ h \end{bmatrix}$$

$$= W \times r \qquad W \times r = \lfloor W_\times \rfloor r \qquad w = \begin{bmatrix} 0 \\ 0 \\ \dot\theta \end{bmatrix} \qquad \dot\theta$$

$|\dot r| = |W|\,|r|\sin\phi = a|\dot\theta|$

$|\dot\theta|$ = angular velocity.

The above case assumes the coordinate from is static.



body frame is rotating at angular velocity w_b

Coordinates of the particle in body frame: $r_B = (x_1, x_2, x_3)^T$

in inertial frame: $r_I(t) = x_1(t)\,i + x_2(t)\,j + x_3(t)\,k = R_{IB} r_B$

R_{IB}, r_B are changing with time. in short, we have $r_I = x_i e_i$.

Derivative: $\dot r_I = R_{IB}\dot r_B + \dot R_{IB} r_B$

$$= R_{IB}\dot r_B + [R_{IB} w_b]_\times r_I$$

$$= R_{IB} V_B + w \times r_I$$

$$V_I = R_{IB} V_B + w \times r_I \Leftrightarrow R_{IB} V_b = V_I - w \times r_I$$

$w = R_{IB} w_b$, is the body frame angular velocity in inertial frame.

Problem:

Inertial frame = static = world frame

Body frame: w, a $\quad a \to v \to$ pose $\quad w \to wt \to Q, R$

accelerometer: $a_m = \dfrac{f}{m} = a - g$

ENU frame: $g = (0, 0, -9.81)^T$

In ENU frame, when IMU is static: $R_{IB} = I$,

$a = 0$, $a_m = -g$.

for free fall: $a = g$, $a_m = 0$.

If IMU has no external force, output should be 0.

but there is a bias: b.

$V_{err} = bat$, $P_{err} = \frac{1}{2}bat^2$.

The bias can be obtained via calibration. However, there's another error, white noise: it's mean is 0, std is σ, a Gaussian process $n(t)$:

$$E[n(t)] = 0,\quad E[n(t_1)\,n(t_2)] = \sigma^2\delta(t_1 - t_2).$$

Above is continuous time, but in reality we sample IMU data:

$$n_d[k] \triangleq n(t_0 + \Delta t) \simeq \frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} n(\tau)\,dt$$

$$E(n_d[k]^2) = E\left(\frac{\sigma^2}{\Delta t}\right), \quad \sigma_d = \frac{\sigma}{\sqrt{\Delta t}}.$$

$\Rightarrow n_d[k] = \sigma_d w[k]$, $w[k] \sim N(0,1)$, $\sigma_d = \dfrac{\sigma}{\sqrt{\Delta t}}$

random walk bias: $\dot b(t) = n(t) = \sigma_b w(t)$, $w(t) = N(0,1)$

$$b_d[k] \triangleq b(t_0) + \int_{t_0}^{t_0+\Delta t} n(t)\,dt$$

$E((b_d[k] - b_d[k-1])^2) = E(\sigma_b^2\Delta t) \Rightarrow b_d[k] = b_d[k-1] + \sigma_{bd} w[k]$.

$w[k] \sim N(0,1)$, $\sigma_{bd} = \sigma_b\sqrt{\Delta t}$.