VIO from scratch 3-2

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$$\arg\min_{q,p,f} \sum_{i=1}^{m}\sum_{j=1}^{n} \| \pi(q_{wc_i}, p_{wc_i}, f_j) - z_{f_j}^{c_i} \|_{\Sigma_{ij}}$$

q: rotation quaternion } camera pose
p: translation
f: feature position in 3D
c_i: ith camera
$\pi()$: projection function.
$z_{f_j}^{c_i}$: observation of f_j in c_i
Σ_{ij}: covariance

Least squares:
find $x^* \in R^n$, to minimize $F(x)$:

$$F(x) = \frac{1}{2}\sum_{i=1}^{m}(f_i(x))^2$$

$m \geq n$, and local minima means $\|x - x^*\| < \delta$, $\Rightarrow F(x^*) \leq F(x)$.

$F(x+\Delta x) = F(x) + J\Delta x + \frac{1}{2}\Delta x^T H \Delta x + O(\|\Delta x\|^3)$

Gradient descent:

We want $F(x_{k+1}) < F(x_k)$

$F(x+\alpha d) \approx F(x) + \alpha J d$, $Jd<0$, we need to find d s.t. $Jd<0$

line search: $\alpha^* = \arg\min_{\alpha>0}\{F(x+\alpha d)\}$

α: step size, d: direction

Steepest GD: $Jd = \|J\|\cos\theta$, when $\theta = \pi$, $d = -J^T$.

Newton:

$$\frac{\partial}{\partial \Delta x}(F(x) + J\Delta x + \frac{1}{2}\Delta x^T H \Delta x) = J^T + H\Delta x = 0$$

$\Delta x = -H^{-1}J^T$.

Damp method:

$F(x+\Delta x) \approx L(\Delta x) \equiv F(x) + J\Delta x + \frac{1}{2}\Delta x^T H \Delta x$

$\Delta x \equiv \arg\min_{\Delta x}\{L(\Delta x) + \frac{1}{2}\mu\Delta x^T\Delta x\}$ $(*)$

$\frac{1}{2}\mu\Delta x^T\Delta x = \frac{1}{2}\mu\|\Delta x\|^2$ is regularization on Δx.

derivative of $(*)$ is $L'(\Delta x) + \mu\Delta x = 0$, $\Rightarrow (H+\mu I)\Delta x = -J^T$

$$f(x) = \begin{bmatrix} f_1(x) \\ \cdots \\ f_m(x) \end{bmatrix}, \quad f^T(x) f(x) = \sum_{i=1}^{m}(f_i(x))^2$$

$$J_i(x) = \frac{\partial f_i(x)}{\partial x}, \quad \frac{\partial f(x)}{\partial x} = J = \begin{bmatrix} J_1(x) \\ \cdots \\ J_m(x) \end{bmatrix}$$

Gauss-Newton:

$f(x+\Delta x) \approx l(\Delta x) \equiv f(x) + J\Delta x$

$$\begin{aligned} F(x+\Delta x) \approx L(\Delta x) &\equiv \frac{1}{2}l(\Delta x)^T l(\Delta x) \\ &= \frac{1}{2}f^Tf + \Delta x^T J^T f + \frac{1}{2}\Delta x^T J^T J\Delta x \quad (*) \\ &= F(x) + \Delta x^T J^T f + \frac{1}{2}\Delta x^T J^T J \Delta x \end{aligned}$$

Set derivative of $(*)$ to zero: $(J^TJ)\Delta x_{gn} = -J^Tf$

$\Rightarrow H\Delta x_{gn} = b$, normal equation.

LM: $(J^TJ + \mu I)\Delta x_{lm} = -J^Tf$, $\mu \geq 0$.

$\mu > 0$, $(J^TJ + \mu I)$ is PSD.

μ large: $\Delta x_{lm} = -\frac{1}{\mu}J^Tf = -\frac{1}{\mu}F'(x)^T$,

μ small: $\Delta x_{lm} \approx \Delta x_{gn}$

$J^TJ \overset{SVD}{\Rightarrow} J^TJ = V\Lambda V^T$, $\Delta x_{lm} = -\sum_{j=1}^{n}\frac{v_j^T F'^T}{\lambda_j + \mu} v_j$

$\mu_0 = \tau \cdot \max\{(J^TJ)_{ii}\}$

if $\Delta x \to F(x)\uparrow$, $\mu\uparrow \to \Delta x\downarrow$

if $\Delta x \to F(x)\downarrow$, $\mu\downarrow \to \Delta x\uparrow$.

Outlier rejection:

robust cost function: $\rho(f^2)$ / M-estimator / IRLS

$$\min_x \frac{1}{2}\sum_k \rho(\|f_k(x)\|^2)$$

let $s_k = \|f_k(x)\|^2$,

$\frac{1}{2}\rho(s) = \frac{1}{2}(\text{const} + \rho'\Delta s + \frac{1}{2}\rho''\Delta^2 s)$