XIVO

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Intro

- The term "robust" in filtering and identification refers to the use of inference criteria that are more robust than L^2 norm. They can be considered special case of Huber functions. where the residual is re-weighted, rather than data selected or rejected. More importantly, the inlier/outlier decision is typically instantaneous.

Notation.

- The spatial frame s is attached to Earth and oriented, so gravity $\gamma^T = [0\ 0\ 1]^T \|\gamma\|$ is known.
- The body frame b is attached to the IMU.
- The camera frame is c.
- The motion is described in the body frame at time t relative to the spatial frame $g_{sb}(t)$. Since the spatial frame is arbitrary, it is co-located with the body at $t=0$.
- To simplify, $g_{sb}(t) \Rightarrow g$, and omit sb.

$$\begin{cases} \dot{T} = v & T(0) = 0 \\ \dot{R} = R(\hat{\omega}_{imu} - \hat{\omega}_b) + n_R \ ? & R(0) = R_0 \\ \dot{v} = R(\alpha_{imu} - \alpha_b) + \gamma + n_v & \\ \dot{\omega}_b = w_b & \\ \dot{\alpha}_b = \xi_b & \end{cases}$$

gravity $\gamma \in \mathbb{R}^3$ is treated as a known parameter.
ω_{imu} are the gyro measurements
ω_b their biases
α_{imu} the accel measurement
α_b their unknown bias
R_0 the unknown initial orientation. of body wrt gravity.

- Initially, we assume points p_i with coordinates $X_i \in \mathbb{R}^3$, $i = 1 \cdots N$ visible from time $t = t_i$ to current time t.
  $\pi: \mathbb{R}^3 \to \mathbb{R}^2$, $X \to [\frac{X_1}{X_3}, \frac{X_2}{X_3}]$ is perspective projection.
- A detector and tracker yields $y_i(t)$, $i = 1 \cdots N$

$$y_i(t) = \pi(g^{-1}(t) p_i) + n_i(t), \quad t \geq 0.$$

where $\pi(g^{-1}(t) p_i)$ is $\dfrac{R_{1:2}^T(t)(X_i - T(t))}{R_3^T(t)(X_i - T(t))}$ with $g(t) \doteq (R(t), T(t))$.

g_{cb} is body frame to camera frame:

$$y_i(t) = \pi(g_{cb}\, g^{-1}(t) p_i) + n_i(t) \in \mathbb{R}^2$$

The unknown, constant parameters p_i and g_{cb} can then be added to the state with trivial dynamics:

$$\begin{cases} \dot{p}_i = 0, \quad i = 1 \cdots N(j) \\ \dot{g}_{cb} = 0 \end{cases}$$

State $x = \{T, R, v, \omega_b, \alpha_b, T_{cb}, R_{cb}\}$. where $g = (R, T)$, $g_{cb} = (R_{cb}, T_{cb})$, and structure parameters p_i are represented in coordinates by

$$X_i = \bar{y}_i(t_i) \exp(\rho_i)$$

which ensures $Z_i = \exp(\rho_i)$ is possible.

- Define the known input $u = \{\hat{\omega}_{imu}, \alpha_{imu}\} = \{u_1, u_2\}$, the unknown input $v = \{w_b, \xi_b\} = \{v_1, v_2\}$. and the model error $w = \{n_R, n_v\}$.
  After defining $f(x)$, $c(x)$, matrix D and $h(x,p) = [\cdots, \pi(R^T(X_i - T))^T, \cdots]^T$ with $p = \{p_1 \cdots p_N\}$,

$$\begin{cases} \dot{x} = f(x) + c(x)u + Dv + c(x)w \\ \dot{p} = 0 \\ y = h(x,p) + n \end{cases}$$

- To enable a smooth estimate, we augment the state with a delay-line, For a fixed interval dt and $1 \leq n \leq k$, define

$$x_n(t) \doteq g(t - n\,dt), \quad x^k \doteq \{x_1 \cdots x_k\}$$

that satisfies

$$x^k(t+dt) \doteq F x^k(t) + G x(t)$$

where

$$F \doteq \begin{bmatrix} 0 & & & \\ I & 0 & & \\ & & \ddots & \\ 0 & & I & 0 \end{bmatrix}, \quad G x(t) \doteq \begin{bmatrix} g(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\mathbf{x} \doteq \{x, x_1 \cdots x_k\} = \{x, x^k\}$.
A k stack of measurements

$$y_j^k(t) = \{y_j(t), y_j(t - dt), \cdots y_j(t - k\,dt)\}$$

can be related to the smoother's state $\mathbf{x}(t)$ by

$$y_j(t) = h^k(\mathbf{x}(t), p_j) + n_j(t)$$

$$h^k(\mathbf{x}(t), p_j) \doteq [h(x(t), p_j)\ \pi(x_1(t) p_j) \cdots \pi(x_k(t) p_j)]^T$$

The overall model is

$$\begin{cases} \dot{x} = f(x) + c(x)u + Dv + c(x)w \\ x^k(t+dt) = F x^k(t) + G x(t) \\ \dot{p}_j = 0 \\ y_j(t) = h^k(\mathbf{x}(t), p_j) + n_j(t), \quad t \geq t_j,\ j = 1 \cdots N(t). \end{cases}$$

- As long as gyro and accel bias rates are not zero, convergence of any inference to a unique estimate cannot be guaranteed.

Robust filtering

- Assume all points appear at time $t=0$, and are present at time t, measurements up to time t as $y^t = \{y(0), \cdots y(t)\}$, Inliers p_j, $j \in J$, $J \subset [1 \cdots N]$, $|J| \ll N$,
- Assume u, v are absent and p_i are known

$$\begin{cases} \dot{x} = f(x) + w \\ y = h(x) + n \end{cases}$$

To determine whether y_i is inlier, consider the event $I \doteq \{i \in J\}$, compute its posterior given all data: $P[I|y^t]$, and compare with alternative $P[\bar{I}|y^t]$, $\bar{I} \doteq \{i \notin J\}$. using the posterior ratio

$$L(i|y^t) \doteq \frac{P[I|y^t]}{P[\bar{I}|y^t]} = \frac{P_{in}(y_i^t | y_{-i}^t)}{P_{out}(y_i^t)} \frac{\epsilon}{1-\epsilon}$$

where $y_{-i} \doteq \{y_j | j \neq i\}$ are all data points but the i-th,
$P_{in}(y_j) \doteq P(y_j | j \in J)$ is inlier density,
$P_{out}(y_j) \doteq P(y_j | j \notin J)$ is outlier density.
$\epsilon \doteq P(i \in J)$ is prior.

- The $P_{in}(y_{J_s}^t)$ for any subset of inlier set $y_{J_s} \doteq \{y_j | j \in J_s \subset J\}$ can be computed recursively at each t:

$$P_{in}(y^t) = \prod_{k=1}^{t} P(y(k) | y^{k-1}).$$

The smoothing state x^t has the property of making future inlier measurements $y_i(t+1)$, $i \in J$ conditionally independent of their past y_i^t: $y_i(t+1) \perp y_i^t\, \mathbf{x}(t)\ \forall i \in J$. and making inliers independent of each other $y_i^t \perp y_j^t | x^t\ \forall i \neq j \in J$.
Using these independence, factors can be computed via standard filtering.

$$P(y(k) | y^{k-1}) = \int P(y(k) | x_k)\, dP(x_k | x_{k-1})\, dP(x_{k-1} | y^{k-1})$$

Starting from $P(y_J(1) | \phi)$, where the density $P(x_k | y^k)$ is maintained by KF.

- Conditioned on a hypothesized inlier set J_{-i}, not containing i, the discriminant $L(i|y^t, J_{-i})$ is

$$L(i|y^t, J_{-i}) = \frac{P_{in}(y_i^t | y_{J_{-i}}^t)\ \epsilon}{P_{out}(y_i^t)\ (1-\epsilon)}$$

can be written as

$$L(i|y^t, J_{-i}) = \frac{\int P_{in}(y_i^t | x^t)\, dP(x^t | y_{J_{-i}}^t)}{P_{out}(y_i^t)} \frac{\epsilon}{1-\epsilon}$$

where $x^t = \{x(0), \cdots, x(t)\}$,
The smoothing density $P(x^t | y_{J_{-i}}^t)$ is maintained by smoother

- The challenge is that we do not know the inlier set J_{-i}:

$$P_{in}(y_i^t | y_{-i}^t) = \sum_{J_{-i} \in P_{-i}^N} P(y_i^t, J_{-i} \cup \{i\} | y_{-i}^t)$$

$$= \sum_{J_{-i} \in P_{-i}^N} P_{in}(y_i^t | y_{J_{-i}}^t) P[J_{-i} | y_{-i}^t]$$

where P_{-i}^N is the power set of $[1 \cdots N]$ not including i.

- To compute the posterior ratio, we have to marginalize J_{-i}, i.e. average over all possible $J_{-i} \in P_{-i}^N$

$$L(i|y^t) = \sum_{J_{-i} \in P_{-i}^N} L(i|y^t, J_{-i})\, P[J_{-i} | y^t]$$