VIO

IMU 以 ≥ 100 Hz 测量角速度/加速度.

IMU $\begin{cases} \text{陀螺仪} \\ \text{加速度计} \end{cases}$

视觉以 15-60 Hz 记录图像/特征点

三维刚体运动 $\begin{cases} \text{世界坐标系 } W \\ \text{IMU 坐标系 } I/B \\ \text{相机坐标系 } C. \end{cases}$

坐标变换: SE(3). e.g. from I to W: T_{WI}

$$T_{WI} = \begin{bmatrix} R_{WI} & t_{WI} \\ 0^T & 1 \end{bmatrix} \in R^{4\times4}$$

v^w 表示速度在 world 系坐标.

四元数:

$$q = [w, x, y, z]^T = [s, v]^T = [\cos\frac{\theta}{2}, \omega\sin\frac{\theta}{2}]^T$$

对时间的导数:

初始四元数 $q=[s,v]$，发生了角轴为 ω,θ，的旋转，Δq.

$$\lim_{\theta\to 0}\frac{q\otimes\Delta q-q}{\theta}=\lim_{\theta\to 0}\frac{[s\cos\frac{\theta}{2}-v^T\omega\sin\frac{\theta}{2},\ s\omega\frac{\theta}{2}+\cos\frac{\theta}{2}v+v\times\omega\sin\frac{\theta}{2}]^T-q}{\theta}$$

$\theta\simeq 0.$

$$=[-\tfrac{1}{2}v^T\omega,\ \tfrac{1}{2}s\omega+\tfrac{1}{2}v\times\omega]^T$$

$$=q\otimes[0,\tfrac{1}{2}\omega]^T$$

$$\therefore\ \dot{q}=q\otimes[0,\tfrac{1}{2}\omega]^T$$

旋转矩阵 R，角速度 ω，R 对于时间导数：

$$\dot{R}=R\omega^\wedge$$

$so(3)$ 导数：优化带旋转函数，计算增量 $\phi\in so(3)$，更新估计值：

$$R\leftarrow R\exp(\phi^\wedge)$$

等价于 $q\leftarrow q\otimes[1,\frac{1}{2}\omega]^T$ (q 归一化)

常见的雅可比：

左扰动.

$$\frac{\partial(Rp)}{\partial\varphi}=\lim_{\varphi\to 0}\frac{\exp(\varphi^\wedge)\exp(\phi^\wedge)p-\exp(\phi^\wedge)p}{\varphi}$$

$$=\lim_{\varphi\to 0}\frac{(I+\varphi^\wedge)\exp(\phi^\wedge)p-\exp(\phi^\wedge)p}{\varphi}$$

$$\lim\quad \varphi^\wedge Rp$$

$$= \lim_{\varphi \to 0} \frac{\quad}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{-(Rp)^\wedge \varphi}{\varphi}$$

$$= -(Rp)^\wedge$$

右扰动.

$$\frac{\partial (Rp)}{\partial \varphi} = \lim_{\varphi \to 0} \frac{R\exp(\varphi^\wedge)p - Rp}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{R(I+\varphi^\wedge)p - Rp}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{R\varphi^\wedge p}{\varphi}$$

$$= \lim_{\varphi \to 0} \frac{-Rp^\wedge \varphi}{\varphi}$$

$$= -Rp^\wedge$$

旋转连乘雅可比: 把矩阵转为向量.

右扰动

$$\frac{d \ln(R_1R_2)}{dR_2} = \lim_{\phi \to 0} \frac{\ln(R_1R_2\exp(\phi^\wedge)) - \ln(R_1R_2)}{\phi}$$

$$= J_r^{-1}(\ln(R_1R_2))^\vee$$

J_r^{-1} 为SO(3)上的右雅可比.

$$\frac{d\ln(R_1R_2)}{dR_1} = \lim_{\phi\to 0}\frac{\ln(R_1\exp(\phi^\wedge)R_2)^\vee - \ln(R_1R_2)^\vee}{\phi}$$

$$= J_r^{-1}(\ln(R_1R_2)^\vee)R_2^T$$

用了SO(3)的伴随性质:

$$R^T\exp(\phi^\wedge)R = \exp((R^T\phi)^\wedge)$$